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GEOMETRY.

247. Proposed by **SETH PRATT, C. E.,** Tecumseh, Neb.

From two given points without a circle to draw two lines meeting in the circumference and making equal angles with the tangent at that point.

Solution by **F. P. MATZ, Sc. D., Ph. D.,** Reading, Pa.

Let C be the center of the circle, T_1PT_2 the tangent to the circle at P; P_1 and P_2 the given points; $PC=r$, $\angle P_1PT_1=\angle P_2PT_2=\phi$, and the polar coördinates of P_1 and P_2, (ρ_1, θ_1) and (ρ_2, θ_2), respectively.

Then $P_1P_2=\sqrt{[\rho_1^2+\rho_2^2-2\rho_1\rho_2\cos(\theta_1-\theta_2)]}$ ....(1).

Also, since $\angle P_1PC=\angle P_2PC=90°+\phi$,

$$P_1P=\sqrt{(\rho_1^2+r^2+2\rho r\sin\phi)}\text{ ....(2), and }P_2P=\sqrt{(\rho_2^2+r^2+2\rho_2 r\sin\phi)}\text{ ....(3).}$$

From the $\triangle P_1PP_2$, $\angle P_1PP_2=180°-2\phi$, we have

$$\cos(180°-2\phi)=-\cos2\phi=\frac{(P_1P)^2+(P_2P)^2-(P_1P_2)^2}{2(P_1P)(P_2P)}\text{ ....(4).}$$

From (4) by means of (1), (2), (3), we have after putting $(\rho_1+\rho_2)=a$, $\rho_1\rho_2=b$, $(\rho_1^2+r^2)=c$, $(\rho_2^2+r^2)=d$, $\rho_1 r=e$, $\rho_2 r=f$, the following equation:

$$\begin{aligned}16ef\sin^6\phi+8(cf+de)\sin^5\phi+4(cd-4ef)\sin^4\phi-8(cf+de)\sin^3\phi\\ -(a^2r^2+4cd+4ef)\sin^2\phi+2[cf+de-ar^3-abr\cos(\theta_1-\theta_2)]\sin\phi\\ +[cd-r^4-b^2\cos^2(\theta_1-\theta_2)-2br^2\cos(\theta_1-\theta_2)]=0\text{ ....(5),}\end{aligned}$$

for the determination of $\sin\phi$.

Knowing $\sin\phi$, the value of P_1P and P_2P can be determined from (2) and (3).

Also solved by J. Scheffer.

248. Proposed by **CHRISTIAN HORNUNG,** Heidelberg University, Tiffin, Ohio.

Given AB, BC in a straight line, to produce it to D so that $AD.CD=BD^2$.

I. Solution by **GRACE M. BAREIS, A. B.,** Bala, Pa.

Describe a circle on AC as diameter, and construct the diameter EF perpendicular to AC. At the point G where EB (or FB) cuts the circle again, draw the tangent. This tangent will cut AC at the required point, D.

For $\angle EGD$ is measured by $\frac{1}{2}(\text{arc}GD+\text{arc}DE)$ and $\angle GBD$ is measured by $\frac{1}{2}(\text{arc}GD+\text{arc}AE)$, since $\text{arc}DE=\text{arc}AE$, $\therefore$ $\triangle GBD$ is isosceles, *i. e.*, $BD=GD$.

But $AD.CD=GD^2$. $\therefore AD.CD=BD^2$.

II. Solution by **R. D. CARMICHAEL,** Hartselle, Ala., and **SAM I. JONES,** Gunter, Texas.

Produce AC indefinitely to H. Lay off $BM=AB$. Find the third proportional to CM and BC. From C measure the distance CD=this third proportional. D is the required point.

$$\text{For } BC^2=CM.CD, \text{ or } BC^2=(AB-BC)CD. \quad \therefore CD.AB=BC^2+BC.CD.$$

$$CD.AB+BC.CD+CD^2=BC^2+2BC.CD+CD^2.$$

$$CD(AB+BC+CD)=(BC+CD)^2.$$

$$AD.CD=BD^2.$$

Also solved by S. A. Corey, J. R. Hitt, F. D. Posey, M. E. Graber, W. W. Landis, and G. W. Greenwood.

249. Proposed by **W. W. BEMAN**, The University of Michigan.

Given the distances of a point in the plane of a square from three of its vertices, to find the side of the square.

250. Proposed by **W. W. BEMAN**, The University of Michigan.

Given the distances of a point in the plane of an equilateral triangle from the vertices; to find the side of triangle. [Perkins' *Geometry*, Olney's *Geometry*.]

I. Solution by **F. D. POSEY**, San Mateo, Calif.

Consider the general case, viz: Given the distances of a point in the plane of a regular n-gon to three consecutive vertices, to find the side of the n-gon.

Let the vertices be A, B, C in order, say clockwise, and P the given point. Let PA, PB, $PC\equiv a$, b, c, respectively. Let $\angle ABP=\alpha$, $\angle PBC=\beta$, taking these clockwise if P be without the angle ABC, and counter-clockwise if P be within. Call the side of the n-gon, x.

There are now two cases: (1) P within the angle ABC of the n-gon, (2) P without this angle. In the first case $\alpha+\beta=\dfrac{n-2}{n}\pi$. $\therefore \cos\beta=-\cos\dfrac{2\pi}{n}\cos\alpha+\sin\dfrac{2\pi}{n}\sin\alpha$. $\therefore \sin\alpha=\operatorname{cosec}\dfrac{2\pi}{n}\cos\beta+\cot\dfrac{2\pi}{n}\cos\alpha$.

In the second case $\alpha+\beta=2\pi-\dfrac{n-2}{n}\pi$. $\therefore \sin\alpha=-\operatorname{cosec}\dfrac{2\pi}{n}\cos\beta-\cot\dfrac{2\pi}{n}\cos\alpha$

Now $\cos\alpha=\dfrac{b^2+x^2-a^2}{2bx}$ (when c is between b and a we have $\cos(2\pi-\alpha)=\cos\alpha$), and $\cos\beta=\dfrac{b^2+x^2-c^2}{2bx}$ (when a is between b and c we have $\cos(2\pi-\beta)=\cos\beta$). In both cases (1) and (2), $\sin^2\alpha+\cos^2\alpha=\left(\operatorname{cosec}\dfrac{2\pi}{n}\cos\beta+\cot\dfrac{2\pi}{n}\cos\alpha\right)^2+\cos^2\alpha=1$, which equation after substituting the above values for $\cos\alpha$ and $\cos\beta$ reduces to:

$$\left[\left(\cot\frac{2\pi}{n}+\operatorname{cosec}\frac{2\pi}{n}\right)^2+1\right]x^4+2\left\{\left(\cot\frac{2\pi}{n}+\operatorname{cosec}\frac{2\pi}{n}\right)\left[\cot\frac{2\pi}{n}(b^2-a^2)\right.\right.$$

$$\left.\left.+\operatorname{cosec}\frac{2\pi}{n}(b^2-c^2)\right]-a^2-b^2\right\}x^2+\left[\cot\frac{2\pi}{n}(b^2-a^2)+\operatorname{cosec}\frac{2\pi}{n}(b^2-c^2)\right]^2$$

$$+(b^2-a^2)^2=0.$$